

Inclusive teaching units in mathematics – Final report

Project context

It is the international agreement of educators and researchers that pupils with special educational needs (SEN) should – as far as possible – be taught together with their non-SEN-classmates in an inclusive setting inside the regular school system, and that such an inclusive setting is beneficiary for both SEN- and non-SEN-pupils. In Austria and the Czech Republic, inclusion has been achieved to varying degrees, though the necessity and extent of special schools is still debated in both countries. In any case, both countries report an increasing number of inclusive classrooms. Support for SEN pupils (and for teachers teaching in inclusive classrooms) ranges from occasional counselling to individual support teachers attending every inclusive lesson. The degree of inclusion also varies widely, from “sitting in the same classroom, but doing something completely different” to the – much preferable – fully integrated participation in the classroom activities. Depending on the available resources, teachers receive support directly in the classroom, but less so in preparation and lesson planning. To that purpose, teachers often see the need to use external resources that are publicly available, e.g. online or at the library. Practice in both countries shows that more attention is paid to pupils with special needs in reading and writing than to pupils with special needs in mathematics. Thus it is quite difficult to find teaching materials for teaching SEN in mathematics classrooms.

Although both countries have been working on support system for teachers (pre- and in-service education, counselling institution, recommendations from psychologists and special educators), many teachers (especially secondary school teachers) still do not report to have sufficient skills when working with SEN pupils. This is also the case of mathematics education where teachers’ training still focuses more on mathematics and different approaches to teaching it than on teaching mathematics in inclusive classrooms as such. Thus, mathematics teachers may feel insecure when actually facing the situation of having one or more SEN in their classroom, wondering what materials may really help and how to plan a good inclusive mathematics lesson.

Project results

The project partners have produced five “ready-made” teaching units and two teaching unit concepts, as described below, and as attached to this report. Those materials have been produced in English and translated into Czech and German.

The project partners have also produced a short theoretical guideline of principles for the development of teaching units for SEN children, as attached to this report.

Details of the project products:

- Five “ready-made” teaching units based on the analysis of the results of international research studies as well as the results of further research studies in the partner countries. These units are specifically designed for inclusive mathematics lessons in lower secondary school. The following teaching units have been produced:
 - Measuring volumes and drinks (8th grade; mathematical topic: concept and calculation of volumes)
 - Food shares (7th grade; mathematical topic: relative and absolute shares, fractions, percentage)
 - Green geometry (6th grade; mathematical topic: perimeter and area of basic geometric figures)

- Volume of cuboids with LEGO Bricks (5th grade; mathematical topic: volumes of geometric bodies)
- Measuring classrooms in different units (7th grade; mathematical topic: basic units of length, decimal numbers)
- Two concepts for further teaching units, allowing teachers to create their own teaching units fitted to the needs of the students in their own classrooms, using research-based concepts. The following concepts have been produced:
 - Measuring (7th grade; mathematical topic: working with decimal numbers, basic principles of measurements)
 - Discounts and sales (7th grade; mathematical topic: basic calculations with percentages)

Project report step-by-step

1. Partners met for the kick-off meeting M1 in Prague, July 1-3, 2024. At this meeting, the proposal and the work plan was revisited, and the next steps were planned. Partners specified the scope of the literature review, worked on the templates to be used for the results, and agreed on the schedule for the next meeting.
2. During July and August 2024, partners reviewed the literature and summarized those aspects that are important for creating the teaching units in a short guidelines paper. They also finalized the templates to be used for describing the teaching units and the concepts.
3. Partners used a scientific conference on September 19-21, 2024, in Pisa, Italy, that they all attended, to meet and discuss the results of the literature analysis, to pre-select potential topics for the teaching units, to plan dissemination activities, and to make final plans for the next official partner meeting M2 in Vienna.
4. Partners met for working meeting M2 in Vienna, October 26-28, 2024. At this meeting, partners made the final decision about which topics to be developed into teaching units, worked on first outlines of these units, and agreed on the principles for their finalization. Partners also discussed already planned and potential further dissemination activities. AT partner set up a shared folder on the University of Vienna's cloud server, for easier sharing of outputs and documents.
5. During November 2024, partners developed pre-final versions of the teaching units in national teams, then sent them for cross-reading and commenting to the other teams. National teams then finalized the teaching units. Partners also developed draft versions of the concepts in national teams.
6. Partners met for the final meeting M3 in Prague, December 16-18, 2024. They discussed and finalized the concepts, presented dissemination results, and discussed further potential dissemination activities (presentation at SEMT conference in summer 2025, workshops for pre- and in-service teachers). Partners had a brainstorming session for a European collaboration (Erasmus+ Key Action 2 cooperation partnership), based on the project results and experiences.

Project dissemination activities

- CZ partners wrote, and Prof. Jarmila Novotná presented, a paper *Principles of adaptation of teaching materials for teaching mathematics in classes with pupils with special educational needs* at the conference *Setkání učitelů matematiky ze všechu typů a stupňů škol* in Srní, Czech Republic, November 7-9, 2024.
- CZ partners gave a teacher training seminar on October 5, 2024, with 23 participants, at Charles University in Prague.
- AT partner gave a project presentation in Newcastle, UK, on November 18, 2024, in the framework of an in-service teacher training activity.

Future collaboration aspects

The two partners plan to be the core for an Erasmus+ Key Action 2 Cooperation Partnership application in the area of School Education. In collaboration with other European universities and with secondary schools, they plan to create and field-test a large variety of full teaching units and teaching unit concepts, and plan courses for pre-service teachers about teaching mathematics to SEN students in an inclusive setting. The potential partner institutions have been informed about the plans on December 23, 2024, and the application plans are currently internally discussed. It is foreseen to submit the application for such a collaboration project with the spring 2025 deadline.

Practical applicability of project results

The results as such are directly applicable, both for teacher trainers (who can use the units and concepts in pre-service and professional development courses) and for school teachers (who can use the concepts and/or the full teaching units in their own classrooms). They are also useful for mathematics educators outside the partnership, where they can either be used “as-is” in teacher training or form the basis of further research activities there.

Financial report

Costs in EUR

Costs of stay (accommodation and subsistence) for Czech staff in Austria

Person	Meeting	Date	Venue	Days	Costs (flat rate)
Jarmila Novotná	M2	26.10.-28.10.2024	Vienna	3	EUR 390.00
Hana Moraová	M2	26.10.-28.10.2024	Vienna	3	EUR 390.00

Costs of travel for Austrian staff to Czech Republic

Person	Meeting	Date	Venue	Days	Costs (train)
Andreas Ulovec	M1	01.07.-03.07.2024	Prague	3	EUR 42.00
Andreas Ulovec	M3	16.12.-18.12.2024	Prague	3	EUR 48.60

Costs in CZK

Costs of stay (accommodation) for Austrian staff in Czech Republic

Person	Meeting	Date	Venue	Days	Costs (real)
Andreas Ulovec	M1	01.07.-03.07.2024	Prague	3	CZK 3,540.00
Andreas Ulovec	M3	16.12.-18.12.2024	Prague	3	CZK 3,200.00

Costs of stay (subsistence) for Austrian staff in Czech Republic

Person	Meeting	Date	Venue	Days	Costs (flat rate)
Andreas Ulovec	M1	01.07.-03.07.2024	Prague	3	CZK 2,700.00
Andreas Ulovec	M3	16.12.-18.12.2024	Prague	3	CZK 2,700.00

Costs of travel for Czech staff to Austria

Person	Meeting	Date	Venue	Days	Costs (train)
Jarmila Novotná	M2	26.10.-28.10.2024	Vienna	3	CZK 1,766.00
Hana Moraová	M2	26.10.-28.10.2024	Vienna	3	CZK 1,113.00

Annex 1 – Teaching units

1. Measuring Volumes and Drinks
2. Food shares
3. Green geometry
4. Volume of cuboids with LEGO Bricks
5. Measuring classrooms in different units



AKTION ÖSTERREICH – TSCHJECHISCHE REPUBLIK
AKTION ČESKÁ REPUBLIKA – RAKOUSKO

Inclusive teaching units in mathematics
Inkluzivní výukové jednotky v matematice

Unit Description

Title	MEASURING VOLUME AND DRINKS
Age of students/grade	8 th grade (14 years old)
Length of the unit	2x45 minutes
Mathematical topic	Measuring, volume
Prerequisite knowledge	<ul style="list-style-type: none">• The concept of volume• Formulas for calculating volume
Materials needed	<ul style="list-style-type: none">• Various containers (e.g., beakers, graduated cylinders, measuring cups)• Irregular objects (e.g., rocks, marbles)• Water• Ruler• Calculator• Graph paper• Different cups and glasses• Photocopies of worksheet (in appendix of this lesson plan)
Objectives	<p>The pupil:</p> <ul style="list-style-type: none">• can determine the volume of different glasses and containers• experiments
Notes on work with special needs children	<ul style="list-style-type: none">• the material supports multisensory learning, it has elements of active learning• the material supports work in groups (mixed ability groups, collaboration of intact and special needs children is recommended)• the material supports use of prior knowledge• The material opens the possibility for pupils to use experiences from everyday life and to apply mathematics in real life context.• The proposed classroom management promotes an increase in both the self-efficacy of pupils with special needs and the recognition by other pupils. A positive consequence may also be an increase in the motivation of pupils with special needs to solve tasks.

Lesson plans:

Lesson 1

Lead in (5 minutes):

Show pupils a container filled partly with water.

Ask your class how much water they think is in the filled container. Elicit how to calculate the volume (formula).

Experiment: Measuring the Volume of Irregular Objects (30 minutes):

1. **Water Displacement Method:** Introduce the water displacement method to measure the volume of irregular objects.
2. **Procedure:**
 - Fill a graduated cylinder with a known volume of water.
 - Carefully place the irregular object into the water.
 - Record the new water level.
 - Calculate the volume of the object by subtracting the initial water volume from the final water volume.
3. **Pupil Experimentation:** Provide each group with irregular objects, graduated cylinders, and water. Have them conduct the experiment and record their observations.
4. **Discussion and Analysis (10 minutes):**
 - **Share Results:** Have each group present their findings and discuss their methods.
 - **Error Analysis:** Discuss potential sources of error in the experiments and how to minimize them.
 - **Critical Thinking Questions:**
 1. How can we measure the volume of objects with irregular shapes?
 2. What are the limitations of the water displacement method?
 3. How can we improve the accuracy of our measurements?

Lesson 2

Warm-up: favourite drinks, whole-class discussion or group work

This can be a whole-class discussion or a simple questionnaire survey. One group may be asking about hot drinks, another group about cold drinks. They decide in the group how they will proceed, how they will organize asking and data processing. This is concluded by presenting the results to the rest of the class.

E.g. 7 pupils prefer Cola, 3 pupils prefer water ...; 2 pupils prefer tea, 8 prefer hot chocolate ...

Activity 1: individual work/pair work

Introducing the situation - reading

WORKSHEET 1

David is opening a new restaurant. He needs to buy beverages from his supplier and he also needs to set their prices the drinks will be sold at. He already bought glasses which have the following proportions: Height: 13 cm, radius: 4 cm. He will use these glasses to sell all the beverages which he has bought in larger containers from his supplier. the beverage-surface will always be 1 cm below the edge of the glass.

See his supplier's offer:

Beverage	Proportions of the cylindrical container	Price per container
Beer keg	Radius 3 dm Height 6 dm	8,000 CZK
Tank of orange juice	Radius 0.5 m Height 1.5 m	14,500 CZK
Pot of hot chocolate	Radius 12 cm Height 12 cm	500 CZK
Pitcher of ice tea	Radius 4.3 cm Height 21 cm	150 CZK
Barrel of lemonade	Radius 2 dm Height 0.8 m	2,400 CZK

What are the volumes of the containers?

At what price does David have to sell glasses of each beverages if he wants to make a profit of at least 58% with each glass?

Record using a table.

(the table may be but may be not given to the pupils – without the table, the difficulty increases)

beverage	Volume of a container [l]	Price per container [CZK]	Amount of glasses in container	Price per glass (without additional charge) [CZK]	Price per glass (with 58% profit) [CZK]	Final price per glass [CZK]
Beer						
Orange juice						
Hot chocolate						
Ice tea						
lemonade						

Solution:

How much beverage is in one glass: $603,19 \text{ cm}^3 \approx 0.6 \text{ l}$

beverage	Volume of a container [l]	Price per container [CZK]	Amount of glasses in container	Price per glass (without additional charge) [CZK]	Price per glass (with 58% profit) [CZK]	Final price per glass [CZK]
Beer	169.6	8000	282	28.37	44.82	45
Orange juice	1178.1	14,500	1963	7.35	11.68	12
Hot chocolate	5.43	500	9	55.56	87.79	88
Ice tea	1.21	150	2	75	118.50	119
lemonade	100.53	2,400	167	14.37	22.70	23

Activity 2: pair work (10 minutes)

Discuss whether the profit of 58% per glass is reasonable. What profit will David make if he sells 100 glasses of beer, 25 glasses of ice tea and 20 cups of tea?

Reflection

1. What have I learnt today?
2. What have I learnt from my classmates?

Appendix 1

WORKSHEET 1 - printable

David is opening a new restaurant. He needs to buy beverages from his supplier and he also needs to set their prices the drinks will be sold at. He already bought glasses which have the following proportions: Height: 13 cm, radius: 4 cm. He will use these glasses to sell all the beverages which he has bought in larger containers from his supplier. the beverage-surface will always be 1 cm below the edge of the glass.

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Beer						
Orange juice						
Hot chocolate						
Ice tea						
lemonade						



AKTION ÖSTERREICH – TSCHECHISCHE REPUBLIK
AKTION ČESKÁ REPUBLIKA – RAKOUSKO

Inclusive teaching units in mathematics
Inkluzivní výukové jednotky v matematice

Unit Description

Title	Food shares
Teaching unit	Calculating relative shares
Age of students/grade	7 th grade (12 – 13 years)
Length of the unit	1 lesson (45 – 50 minutes)
Mathematical topic	Relative shares, percentage
Prerequisite knowledge	<ul style="list-style-type: none"> • Relative and absolute shares • Fractions • Percentage
Materials needed	<ul style="list-style-type: none"> • Simple food recipe (the given one in the worksheet, or a recipe that is suggested by a pupil) • Ingredients of the recipe • Bread (if the given recipe on the worksheet is used) • Scales, Tablespoons, Teaspoons
Objectives	<p>The pupil:</p> <ul style="list-style-type: none"> • Can calculate relative shares out of (given or measured) absolute shares, and express these shares either as a fraction or a percentage • Can convert units of mass • Can measure the weight of food components
Notes on work with special needs children	<ul style="list-style-type: none"> • The material supports multisensory learning • The material supports work in groups (mixed ability groups, collaboration of non-special needs and special needs children is recommended) • The material supports use of prior knowledge • The material opens the possibility for pupils to use experiences from everyday life • The opinion of every pupil is asked in Task 3. Pupils should learn that everyone's opinion, regardless of their special needs, is heard and counts the same. • Eventual food allergies, as well as personal, cultural or religious food restrictions, have to be clarified beforehand • If a recipe is chosen that requires cooking or baking, this would need to be pre-arranged; cold dishes would be easier to handle

Lesson plan

- The teacher introduces the topic of the lesson (calculating relative shares if the parts are given in different units).
- The teacher introduces an example:

Hot Chocolate recipe:

- 200 Gram milk
- 2 Teaspoons cocoa powder (unsweet, i.e. pure cocoa)
- 1 Tablespoon sugar



Boris loves hot chocolate but is vigilant about his sugar intake. He wants to know whether the hot chocolate made with this recipe contains more than 10 % sugar.

First, we have to ask ourselves: 10 % of what? A percentage does only make sense in reference to a whole. Here, the “whole” would be the whole drink, i.e. the sum of all ingredients. But what unit do the 10 % relate to? 10 % of the volume? 10 % of the mass? 10 % of the pieces that we add? In foods, the most-used unit is mass. To find out whether the sugar is more than 10 % of the mass of the whole drink, we first have to calculate the total mass of the whole drink. This can be easily done by adding the mass of all ingredients. However, as we can see in the recipe, only one ingredient is given with its mass (in gram), the others are given in other measurements. So the first thing we need to do is convert all those ingredients into measurements of mass. We can do this by simply weighing them on a scales.

The teacher demonstrates the weighing of cocoa powder on a scales. He/she can then either also weigh the sugar or ask a pupil to do so. The results are written on the board. Here are some plausible results as an example.

- 200 Gram milk
- 2 Teaspoons cocoa powder = 5 Gram cocoa powder
- 1 Tablespoon sugar = 15 Gram sugar

Adding these values, we get the total mass of all ingredients: 220 g. The relative share of the sugar can now be calculated by dividing the mass of the sugar by the total mass of the drink, i.e.

$$\text{Relative share of sugar} = \frac{15}{220} \approx 0.068 = 6.8 \%$$

The share of the sugar is below 10 %.

If the facilities in the school allow it, the hot chocolate can of course be actually prepared and consumed.

- The teacher divides the class into groups of three to four pupils. Each group gets a scales, a tablespoon, a teaspoon, and the ingredients for the recipe (if the recipe given in the worksheet is used, each group also gets a few slices of bread).
- Pupils work in groups on Tasks 1 and 2 from the worksheet.
- Pupils are asked to discuss about Task 3 in the groups.
- The groups share their opinions of Task 3 to the whole class.

Worksheet – Calculating relative shares in food recipes

Background

Food recipes list a number of ingredients. They are often given in different units, e.g. in Gram, Liter, spoons etc. To calculate the relative share of one of the ingredients, we need the mass of all the ingredients to be in the same unit.

Simple paprika spread:

- 250 Gram curd
- 100 Gram soft butter or margarine
- 1 Tablespoon paprika powder
- 1 Teaspoon salt
- 1 Teaspoon caraway



Mix all ingredients. Let stand for 10 minutes and stir the spread again.

Task 1:

Calculate the relative share of paprika in this paprika spread! Is it more than 5 %?

Task 2:

If you would use double as much paprika, would the relative share of paprika be doubled as well?
Why, or why not?

Task 3:

Make the spread according to the recipe, put it on a piece of bread, and taste it! Would you change something in the recipe? E.g. more paprika, less salt, or no caraway? Or add something else?



AKTION ÖSTERREICH – TSCHECHISCHE REPUBLIK
AKTION ČESKÁ REPUBLIKA – RAKOUSKO

Inclusive teaching units in mathematics
Inkluzivní výukové jednotky v matematice

Unit Description

Teaching unit	Green Geometry
Age of students/grade	12 years
Length of the unit	6 x 45 minutes
Mathematical topic	Figures in the plane
Prerequisite knowledge	<ul style="list-style-type: none"> • Basic geometric figures • Perimeter and area of basic geometric figures
Materials needed	<ul style="list-style-type: none"> • Task sheet 1 • Task sheet 2 • Info sheet for the teacher • Various gardening tools and materials (optional, if possible)
Objectives	<p>The pupil</p> <ul style="list-style-type: none"> • can apply knowledge about properties of basic geometric figures (particularly triangle, rectangle, isosceles trapezoid, and circle) in practical, real-life situations • can apply the Pythagorean Theorem in real-life contexts • can use measurements and convert units in real-life contexts
Notes on work with special needs children	<ul style="list-style-type: none"> • the material supports multisensory learning, it has elements of active learning • the material supports work in groups (mixed ability groups, collaboration of intact and special needs children is recommended) • the material supports use of prior knowledge • The material opens the possibility for pupils to use experiences from everyday life and to apply mathematics in real life context. • The field project allows hands-on, practical experience. • The proposed classroom management promotes an increase in both the self-efficacy of pupils with special needs and the recognition by other pupils. A positive consequence may also be an increase in the motivation of pupils with special needs to solve tasks.
Comments	<ul style="list-style-type: none"> • The unit is designed in a way that it includes field work and actual planting.

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| | <ul style="list-style-type: none">• The teacher must clarify with the school or municipality which green space in the city can be used by the pupils before the teaching units are realised. To avoid road traffic, it would be best to choose a green space in a park or playground. Above all, the location and size must be clarified, as well as the financial support from the municipality (costs incurred for e.g. soil, stones, gravel, bench, plants, etc.). The maintenance of the green space (especially watering) must also be clarified with the municipality. The harvesting of the selected vegetables (e.g. peas or pumpkin) should be left to the pupils. The teacher is responsible for ordering the flowers and plants as well as the required materials (e.g. gravel and stones) in co-operation with the local authority.• The teacher must look at the green space on-site in advance and take concrete measurements in order to plan the lessons. In the following suggested teaching units, a sample solution is worked out using a rectangular green space measuring 25 m x 8 m as an example. The actual size and design may vary greatly from the sample solution depending on the circumstances of the area and the ideas of the pupils. |
|--|--|

Lesson plan:

Lesson 1 (45 minutes)

Time	Method	Content and comments
ca. 25 min	Partner work, then comparison in whole-class setting	<p>The lesson starts with a general overview of (selected) plane figures and formulas for determining and area. (see task 1)</p> <p>Note: The pupils are already familiar with the plane figures mentioned, and perimeters and areas have already been calculated, but only with specific figures given in the textbook. The upcoming project is for their practical application.</p> <p>Task 1 is solved to revise the concepts already known.</p>
ca. 20 min	Exposition (explanation of project), Brainstorming, whole-class discussion	<p>The teacher presents pictures of gardens and introduces the pupils to the upcoming project. (See 'Presentation by the teacher')</p> <p>At the same time, the geometric figures mentioned above are taken into account. The teacher explains that the pupils will be given a rectangular green area in the city (as an example we use 25 m x 8 m). The pupils should already make suggestions for the design of this green space (separating and creating flower and vegetable beds, selecting plants, creating a footpath, possibly planning a bench, etc.). The pupils must be made aware that they should always incorporate the geometric figures just discussed (from task 1) when planning.</p> <p>Together consider what equipment the pupils will need for the inspection in the following lesson (tape measure, paper and pencil for sketches, possibly a calculator and tablet).</p>

Lesson 2 (45 minutes)

Time	Method	Content and comments
45 min	Expedition, group work	<p>The pupils and the teacher inspect the chosen location on-site. The pupils first take the most important measurements together. (see Sketch 1 as an example). The dimensions may of course differ from the proposed sketch. Accordingly, they must take their own dimensions into account when making the calculations.</p> <p>The pupils are then divided into 2 groups:</p> <ul style="list-style-type: none">- Group 1 considers the planting (aesthetic preferences)- Group 2 considers the detailed planning of the area (footpath, flower beds, required materials such as gravel, soil, crushed stone, stones)

Lesson 3 (45 minutes)

Time	Method	Content and comments
ca. 20 min	whole-class work	<p>The dimensions of the green space are drawn on a sheet of paper. Each pupil is to draw a sketch on their own sheet of paper, with one pupil taking it in turns to draw/add to the sketch on the board (for an example see sketch 2).</p>
ca. 25 min	whole-class work	<p>The ideas for designing the green space from the last lesson are then presented. The pupils can show the relevant plants or required materials on the beamer.</p>

Lesson 4-5 (90 minutes)

Time	Method	Content and comments
90 min	Partner work, then comparison of results in whole-class setting	Calculations of area and perimeter in a real-life context; applications of Pythagorean theorem; calculations of the number of pieces or quantity of materials required (see task 2)

If possible, the practical realization

Lesson 6-7 (90 minutes)

Time	Method	Content and comments
90 min	Group work	Realisation of all planned activities; Planting the beds, laying out the paths, ...

Task sheet 1

Task 1:

Fill in the gaps in the table correctly! Use the following equations.
Fill in the missing names and figures yourself!

$$A = a \cdot h_a = b \cdot h_b$$


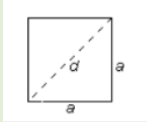
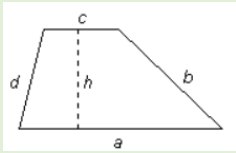
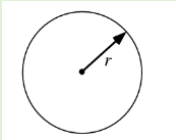
$$U = 2 \cdot (a + b)$$

$$A = a^2$$

$$A = \frac{(a+c) \cdot h}{2}$$

$$U = 2 \cdot r \cdot \pi$$

$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$

Geometric figure	Sketch	Perimeter	Area
Rectangle			$A = a \cdot b$
		$U = 4 \cdot a$	
Parallelogram		$U = 2 \cdot (a + b)$	
Triangle		$U = a + b + c$	
		$U = a + b + c + d$	
Circle			$A = r^2 \cdot \pi$

Task sheet 1 – solutions

Task 1:

Fill in the gaps in the table correctly! Use the following equations.
Fill in the missing names and figures yourself!

$$A = a \cdot h_a = b \cdot h_b$$

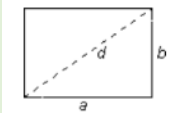
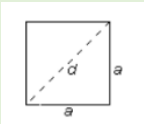
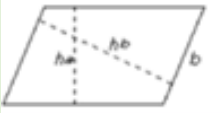
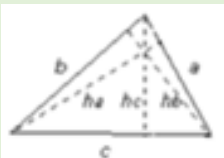
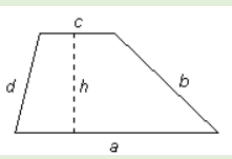

$$U = 2 \cdot (a + b)$$

$$A = a^2$$

$$A = \frac{(a+c) \cdot h}{2}$$

$$U = 2 \cdot r \cdot \pi$$

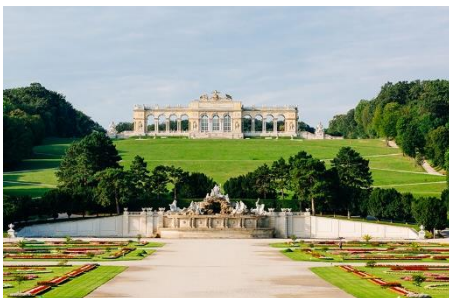
$$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$$

Geometric figure	Sketch	Perimeter	Area
Rectangle		$U = 2 \cdot (a + b)$	$A = a \cdot b$
Square		$U = 4 \cdot a$	$A = a^2$
Parallelogram		$U = 2 \cdot (a + b)$	$A = a \cdot h_a = b \cdot h_b$
Triangle		$U = a + b + c$	$A = \frac{a \cdot h_a}{2} = \frac{b \cdot h_b}{2} = \frac{c \cdot h_c}{2}$
Trapezoid		$U = a + b + c + d$	$A = \frac{(a + c) \cdot h}{2}$
Circle		$U = 2 \cdot r \cdot \pi$	$A = r^2 \cdot \pi$

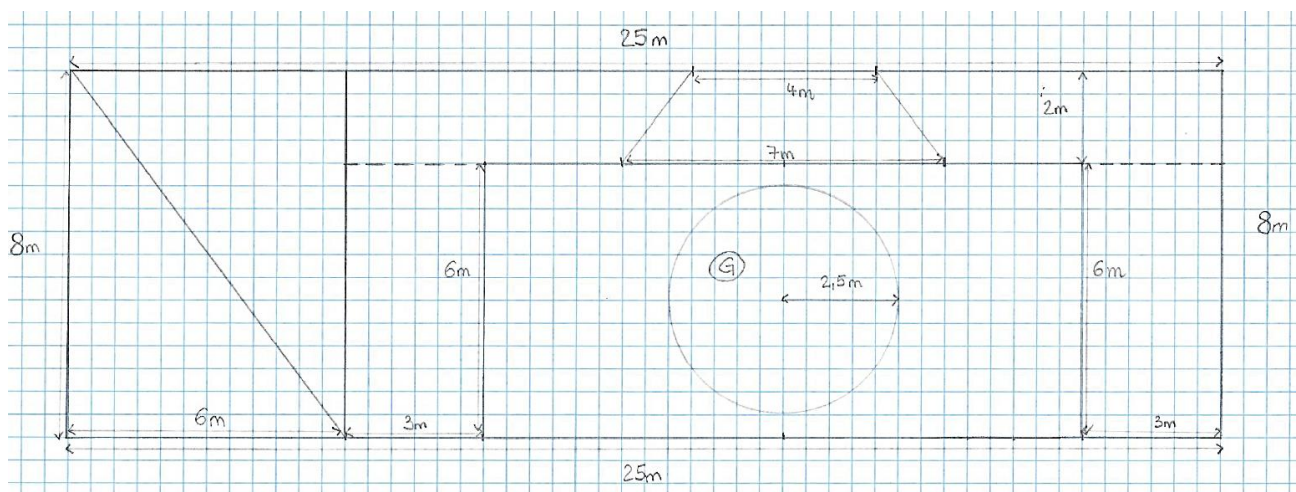
Info sheet for the teacher

Presentation by the teacher:

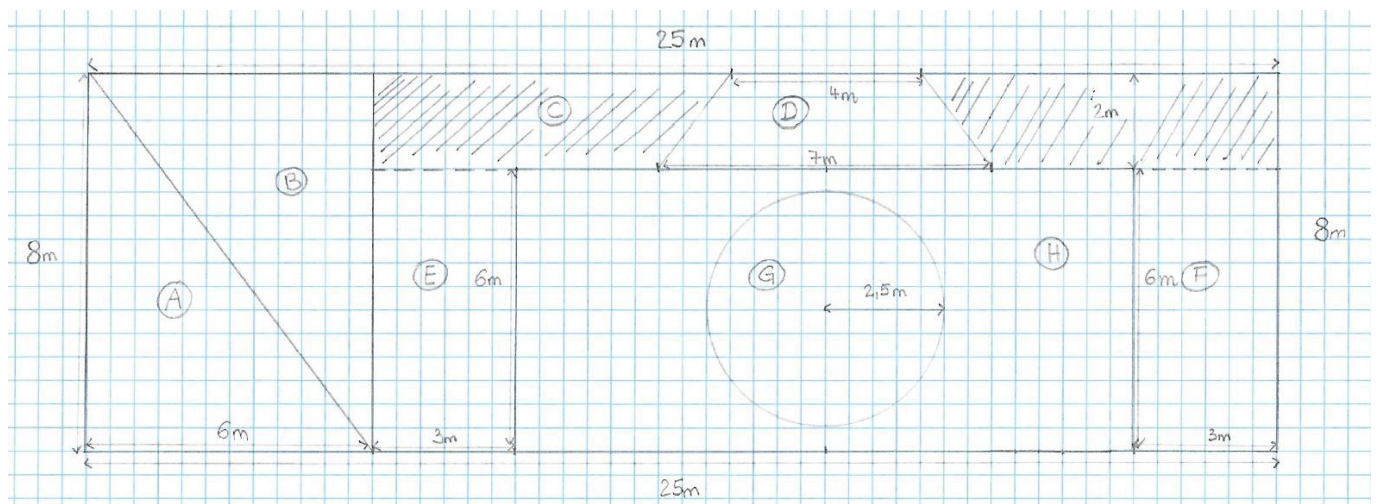
An example of a famous garden is the garden in front of Schönbrunn Palace and the Gloriette in Vienna.



Sketch 1



Sketch 2



Sketch 2 shows a detailed plan (see the figure for corresponding dimensions):

- Areas A and B are triangles of equal area
- Area C is the hatched area (which is excluded by an isosceles trapezoid)
- Area D is an isosceles trapezoid
- Areas E and F are rectangles of equal area
- Area G is a circle
- Area H is a part of a rectangle, which is excluded from the circular area.

The green area has a size of 25 m x 8 m.

Task sheet 2

Task 2:

- a) Area A: Pumpkins are to be grown.

Calculate the corresponding surface area of the bed.

Think about a pumpkin variety and research on the internet how much space a pumpkin plant needs. How many plants can be planted?

Note: The pumpkin plant is chosen as an example here because it is very popular and well-known in Austria and can be grown in spring and harvested in autumn (so the summer holidays don't matter).



- b) Area B: Peas are to be grown.

You need 20-25 g of pea seed per square metre.

Calculate how many grams of seed you need for bed B!



- c) A path made of old bricks is to be laid between beds A and B.
Calculate the length of the path and give a way of laying the bricks.
Calculate the number of bricks required!



- d) Area D represents an isosceles trapezoid and is to be designed with gravel so that, for example, a bench can be placed there.
How many kg of gravel are needed if around 6kg of gravel are required for 1 m²?

- e) Area C is to be planted with a total of 4 hardy trees.
The entire area should then be filled with wood chips.

As a group, consider which hardy trees (maximum width of 3 metres) are suitable!
Calculate the size of the area!



- f) The areas E and F should be designed with hardy plants. Calculate that one plant requires approximately 120 cm x 120 cm of space.

Calculate how many plants can be planted.

Think about which plants can be used so that different (hardy) plants bloom continuously from March to October. The height of the plants should be approx. 50 to 150 cm.

Take into account the different popular and well-known plants in different countries! Create a table of the plants with growth height, flowering time and picture of the plant!

- g) The circular bed is to be planted with edible berries.

Calculate the area and select three types of berries to be planted.

The circular bed should also be surrounded with old bricks (dimensions 250 mm x 120 mm).

Calculate how many bricks are needed!



- h) Lawn is to be grown on the remaining area H.
Calculate the area and find out how much lawn seed is needed if 30 g per square metre is required!



Finally, the sketch should be revised again and refined with specific colours and/or pictorial representations.

Task sheet 2 – (examples of) solutions

Task 2:

- a) Area A: Pumpkins are to be grown.



Calculate the corresponding surface area of the bed.

Think about a pumpkin variety and research on the internet how much space a pumpkin plant needs.
How many plants can be planted?

Note: The pumpkin plant is chosen as an example here because it is very popular and well-known in Austria and can be grown in spring and harvested in autumn (so the summer holidays don't matter).

$$A = \frac{6 \cdot 8}{2} = 24 \text{ m}^2$$

According to an Internet research, a Hokkaido-pumpkin-plant needs approx. 2.5 m^2 room.

$$24 / 2.5 = 9.6$$

One can grow up to 9 pumpkin plants.

Task 2:

- b) Area B: Peas are to be grown.



You need 20-25 g of pea seed per square metre.

Calculate how many grams of seed you need for bed B!

$$A = \frac{6 \cdot 8}{2} = 24 \text{ m}^2$$

$$24 \cdot 20 = 480 \text{ g}$$

$$24 \cdot 25 = 600 \text{ g}$$

One needs between 480 g and 600 g of seeds.

Task 2:

- c) A path made of old bricks is to be laid between beds A and B.
Calculate the length of the path and give a way of laying the bricks.
Calculate the number of bricks required!



$$\text{Length of the path} = \sqrt{8^2 + 6^2} = 10$$

The path is 10 m long.

If a typical old brick measuring 250 mm x 120 mm is used, the bricks could be laid out lengthwise (2 rows in parallel would then be needed, so as the path not being too narrow).

Per row one gets: $\frac{10\text{ m}}{0.25\text{ m}} = 40$ pieces of brick

For two rows, one would need 80 bricks.

Task 2:

- d) Area D represents an isosceles trapezoid and is to be designed with gravel so that, for example, a bench can be placed there.
How many kg of gravel are needed if around 6kg of gravel are required for 1 m^2 ?

$$A = \frac{(7+4) \cdot 2}{2} = 11\text{ m}^2; 11 \cdot 6 = 66\text{ kg}$$

One needs approximately 66 kg of gravel.

Task 2:

- e) Area C is to be planted with a total of 4 hardy trees.
The entire area should then be filled with wood chips.

As a group, consider which hardy trees (maximum width of 3 metres) are suitable!
Calculate the size of the area!

Winter-hardy (narrow) small trees include, for example, the Photinia (*Photinia × fraseri*, left) or the harlequin willow (*Salix integra*, right).



$$A = 19 \cdot 2 - \frac{(7+4) \cdot 2}{2} = 27\text{ m}^2$$

Task 2:

- f) The areas E and F should be designed with hardy plants. Calculate that one plant requires approximately 120 cm x 120 cm of space.

Calculate how many plants can be planted.




Think about which plants can be used so that different (hardy) plants bloom continuously from March to October. The height of the plants should be approx. 50 to 150 cm.

Take into account the different popular and well-known plants in different countries! Create a table of the plants with growth height, flowering time and picture of the plant!

$$A = 120 \cdot 120 = 14400\text{ cm}^2 = 1.44\text{ m}^2 \text{ per plant}; A_{\text{available}} = 3 \cdot 6 \cdot 2 = 36\text{ m}^2; \frac{36}{1.44} = 25$$

One can plant approximately 25 plants.

Potential plants:

Plant	Growth height	Flowering time	Picture
Euphorbia (e.g. <i>Euphorbia epithymoides</i>)	Ca. 50 cm	April-July	
Spiraea (e.g. <i>Spiraea japonica</i>)	Ca. 50 cm - 150 cm	July-August	
Aster (e.g. <i>Symphyotrichum novae-angliae</i>)	Ca. 140 cm	September-October	

Task 2:

- g) The circular bed is to be planted with edible berries.

Calculate the area and select three types of berries to be planted.
The circular bed should also be surrounded with old bricks
(dimensions 250 mm x 120 mm).
Calculate how many bricks are needed!



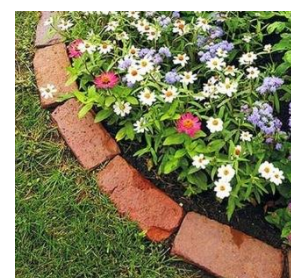
$$A = 2.5^2 \cdot \pi \approx 19.6 \text{ m}^2$$

Potential berries: raspberry, blackberry, blueberry.

$$U = 2 \cdot 2.5 \cdot \pi \approx 15.7 \text{ m}$$

$$\frac{15.7}{0.25} = 62,8$$

One needs approximately 63 bricks to surround the bed.



Task 2:

- h) Lawn is to be grown on the remaining area H.
Calculate the area and find out how much lawn seed
is needed if 30 g per square metre is required!



$$A = 13 \cdot 6 - 19.6 = 58.4 \text{ m}^2$$

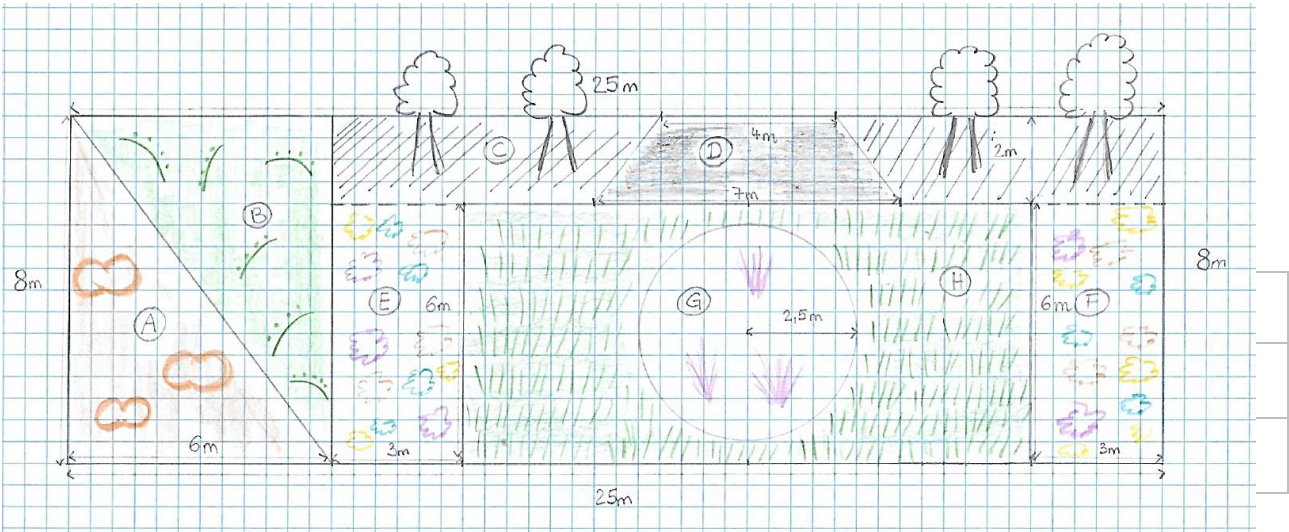
$$58.4 \cdot 30 = 1752 \text{ g} \approx 1.8 \text{ kg}$$

One needs approximately 1.8 kg lawn seed.

Task 2:

- i) Finally, the sketch should be revised again and refined with specific colours and/or pictorial representations.

A new sketch may e.g. look like this:





AKTION ÖSTERREICH – TSCHECHISCHE REPUBLIK
AKTION ČESKÁ REPUBLIKA – RAKOUSKO

Inclusive teaching units in mathematics
Inkluzivní výukové jednotky v matematice

Unit Description

Title	Volume of cuboids with LEGO Bricks
Teaching unit	Calculating volumes of cuboids
Age of students/grade	5 th grade (10 – 11 years)
Length of the unit	1 lesson (45 – 50 minutes)
Mathematical topic	Volumes of geometric bodies
Prerequisite knowledge	<ul style="list-style-type: none"> • Working with decimal numbers • Basic principles of measurements
Materials needed	<ul style="list-style-type: none"> • Standard 4x2 LEGO bricks • Various boxes in the form of cuboids (large enough so that several LEGO bricks fit inside, and small enough so that the pupils can easily lift and carry them; examples: empty snack box, empty shoe box, box for chalks ...)
Objectives	<p>The pupil:</p> <ul style="list-style-type: none"> • Can use LEGO bricks to build cuboids of various sizes • Can count the LEGO bricks and multiply them with the volume of one brick to calculate the volume of the cuboids • Eventually: Can use the term “length x width x height” to calculate the volume of a cuboid
Notes on work with special needs children	<ul style="list-style-type: none"> • The material supports multisensory learning • The material supports work in groups (mixed ability groups, collaboration of non-special needs and special needs children is recommended) • The material supports use of prior knowledge • The material opens the possibility for pupils to use experiences from everyday life and childhood play • The proposed work with LEGO bricks can increase the ability for precise working with hands

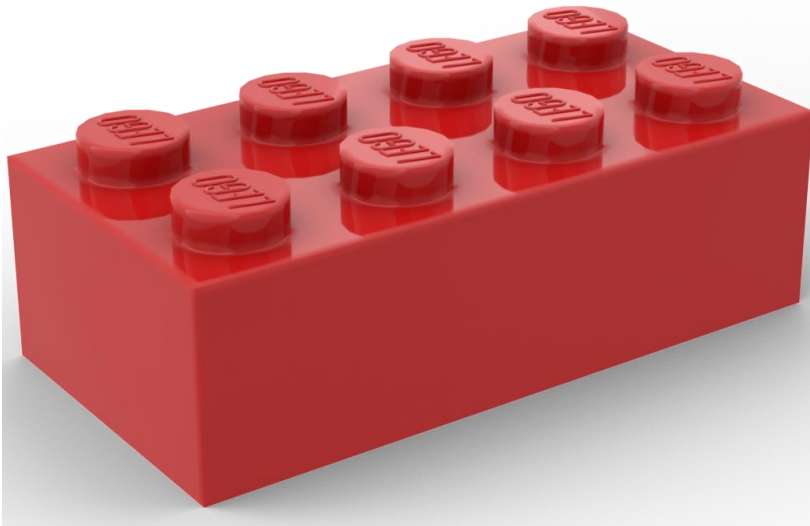
Lesson plan

- The teacher introduces the topic of the lesson (measuring volumes).
- The teacher introduces Example 1. He/she either builds a $2 \times 2 \times 1$ cuboid with LEGO bricks or lets a pupil build it. He/she then completes Example 1.
- The teacher divides the class into groups of three to four pupils. Each group gets a sufficient number of LEGO bricks and a copy of the worksheet
- Pupils work in groups on Task 1 from the worksheet.
- After Task 1, depending on the abilities of the pupils, the groups can either read the paragraph after Task 1 on the worksheet themselves, or the teacher explains the content of this paragraph.
- The pupils work on Task 2.
- The teacher distributes boxes, one for each of the groups. The boxes do not need to be of the same size.
- The pupils work on Task 3 and Task 4.
- Those groups who have finished Task 4, as checked by the teacher, can continue to “Another way” on the worksheet.
- Time and ability of the pupils permitting, all or some groups can work on Task 6.
- The teacher coordinates and monitors group work, providing help when needed.

Worksheet – Using LEGO Bricks for measuring volume

Background

What we have already learned: “Measuring” (by using units) means to see how often a unit fits into that you want to measure. If you want to measure the volume of a box, you have to see how often a volume unit fits into this box. In this lesson, we will use 4x2 LEGO bricks as a volume unit.



Example 1:

We can build a cuboid that is 2 LEGO bricks long, 2 LEGO bricks wide, and 1 LEGO brick high.

We now count the number of LEGO bricks that we have used. The result is 4. The volume of this cuboid is 4 LEGO bricks.

Task 1:

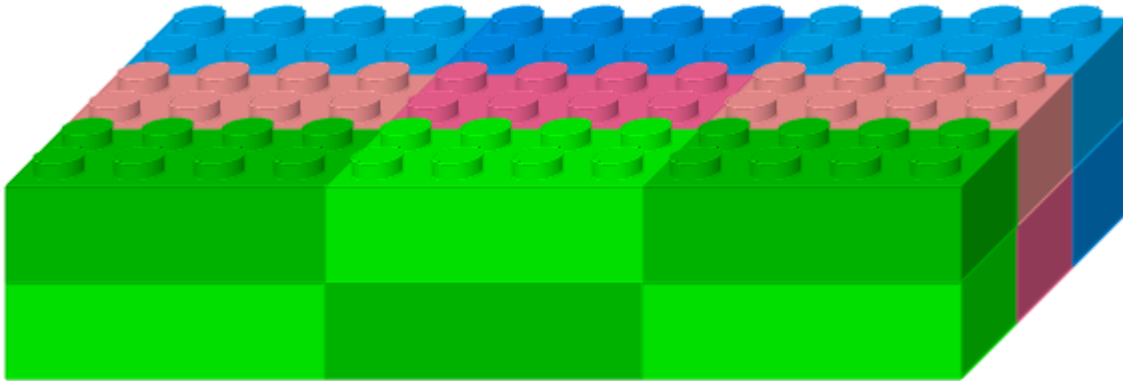
Build a cuboid that is 4 LEGO bricks long, 3 LEGO bricks wide, and 2 LEGO bricks high. Count the number of LEGO bricks that you have used! This gives you the volume of this cuboid (measured in LEGO bricks).

Now of course “LEGO bricks” is not a very usual unit that is used to measure volume. One possible unit would be cubic centimetres (cm³). 1 LEGO brick has a volume of 5.75 cm³. The cuboid from Example 1 had a volume of 4 LEGO bricks, so its volume in cm³ is $4 \cdot 5.75 \text{ cm}^3 = 23 \text{ cm}^3$.

Task 2:

Calculate the volume in cm³ of the cuboid you have built for Task 1!

Can we use LEGO bricks to find out the volume of a box, at least approximately? We can do that if we build a cuboid out of LEGO bricks that has (approximately) the size of the box and then again count the LEGO bricks.



Task 3:

Build a cuboid out of LEGO bricks that has approximately the same size as the box you have just received! Decide as a group whether the cuboid should be a little larger or a little smaller than the box!

Task 4:

Count the number of LEGO bricks that you have used for the cuboid in Task 3! If the cuboid is of similar size as the box, this gives you a good estimate for the volume of the box measured in LEGO bricks.

Another way ...

Counting large numbers of LEGO bricks can be tedious! We can also find out the volume of a cuboid by multiplying its length, width, and height – measured in LEGO bricks!

Example 2:

The cuboid in Example 1 was 2 LEGO bricks long, 2 LEGO bricks wide, and 1 LEGO brick high. Its volume is therefore $2 \cdot 2 \cdot 1 = 4$ LEGO bricks.

Task 5:

Use the same method to calculate (rather than count) the number of LEGO bricks in Task 4!

Task 6:

Use the result of Task 5 to calculate the volume of the cuboid in cm^3 !



AKTION ÖSTERREICH – TSCHECHISCHE REPUBLIK
AKTION ČESKÁ REPUBLIKA – RAKOUSKO

Inclusive teaching units in mathematics
Inkluzivní výukové jednotky v matematice

Unit Description

Title	Measuring classroom in different units
Teaching unit	Measuring and conversions
Age of students/grade	7 th grade (12 – 13 years)
Length of the unit	1 lesson (45 – 60 minutes)
Mathematical topic	Units of length and their conversion
Prerequisite knowledge	<ul style="list-style-type: none"> • Basic units of lengths, decimal numbers
Materials needed	<ul style="list-style-type: none"> • Various measuring instruments (rulers, tape measure, string)
Objectives	<p>The pupil:</p> <ul style="list-style-type: none"> • uses tools for measuring • converts units (SI units as well as less usual units) • uses decimal numbers • cooperates in a group • presents results of their work
Notes on work with special needs children	<ul style="list-style-type: none"> • the material supports multisensory learning • the material supports work in groups (mixed ability groups, collaboration of intact and special needs children is recommended) • the material supports use of prior knowledge • The material opens the possibility for pupils to use experiences from everyday life • The proposed classroom management based on group work promotes an increase in both the self-efficacy of pupils with special needs children and the recognition by other pupils. A positive consequence may also be an increase in the motivation of pupils with special needs to solve tasks.

Lesson plan

- The teacher introduces the topic of the lesson (Length measurement and units of length).
- The teacher introduces the task, divides the class into groups of three to four pupils
- Each group gets a copy of the worksheet, together they read the first paragraph of the instructions and the first task, the teacher clarifies any unclear points.
- Pupils work in groups on tasks 1 and 2 from the worksheet. The teacher coordinates and monitors group work, providing help when needed.
- Pupils present and share the results of the measurements.
- Whole group and group work – task 3 (1 person from each group participates in measuring the classroom with a tape measure, then in the group compare with what they measured in the groups, how accurate they were)
- If there is time left, tasks 4 and 5, pair work or group work

Worksheet - Length and its measurement

Background

Length is one of the fundamental physical quantities. It indicates the dimension of bodies (length, width, height, depth) or the distance between two points in space. The oldest units of length were derived from the sizes of human body parts: thumb, palm, fist, foot, elbow, step, and others.

Different units were used in different parts of the world. Over time, there was a need to unify the units of length, and so the metre was defined. After several changes, since 1983 the metre is defined as the distance travelled by light in $\frac{1}{299\,792\,458}$ seconds.

Task 1:

Using your own bodies in the group, choose three units of length suitable for measuring the dimensions of the classroom and use them to measure it. You are not allowed to use any measuring devices other than your body parts. Record the data in the table, think of a name as well as a symbol for each unit and write them in the table.

	Name and symbol of each unit		
Length of the classroom			
Width of the classroom			

Task 2:

Using a suitable measuring device, convert the data in your own units to metres and record them in the following table. Reflect on the variation and accuracy of the calculated data. Also record the conversion relationship between your units and metres, e.g. 1 arm = 0.75 m.

	Measurement 1	Measurement 2	Measurement 3
Length of the classroom			
Width of the classroom			

Task 3:

Use the tape measure to measure the dimensions of the classroom in metres and compare them with the figures in the table in task 2.

Task 4 (optional extension):

Even today, some countries use units of length other than metres. This is particularly the case in the UK and the USA. Although the UK officially adopted the metric system in 1995, some of the original units are still in use today, not only by the population but also officially. The following table lists the original British units and the relationships between them and metric units. Using a calculator, complete it by converting each unit to metres.

unit	symbol	conversion relationship	conversion to metres
inch	in, "	1 in = 2,54 cm	
foot	ft, '	1 ft = 12 in	
yard	yd	1 yd = 3 ft	
furlong	fur	1 fur = 660 ft	
mile	m, mi	1 mi = 1760 yd	

Task 5 (optional extension):

Look again at the dimensions of the classroom you have measured. Choose the ones you think are most accurate, write them down in the table and convert them to the given units.

	in	ft	yd	mi
length				
width				

Annex 2 – Teaching unit concepts

1. Measuring
2. Discounts and sales



**AKTION ÖSTERREICH – TSCHECHISCHE REPUBLIK
AKTION ČESKÁ REPUBLIKA – RAKOUSKO**

**Inclusive teaching units in mathematics
Inkluzivní výukové jednotky v matematice**

Concept of teaching material

Title	Measuring
Teaching unit	Measuring and conversion
Age of students/grade	7 th grade (12 – 13 years)
Length of the unit	1 – 2 x 45 minutes
Mathematical topic	Units of length, area, volume, or mass, and their conversion
Prerequisite knowledge	<ul style="list-style-type: none"> • Working with decimal numbers • Basic principles of measurements
Materials needed	<ul style="list-style-type: none"> • Depending on what is measured; either regular measuring instruments (rulers, tape measure, scales, measuring cups, ...), or everyday life objects that can be used for measuring (string, cups, spoons, bottles, toy building blocks, sheets of paper, ...)
Objectives	<p>The pupil:</p> <ul style="list-style-type: none"> • uses measuring instruments or other objects for measuring • realises that the result of a measurement consists of both a number and a unit of measurement (or a unit symbol) • converts units (SI units as well as less usual units) • uses decimal numbers • cooperates in a group • presents results of their work
Notes on work with special needs children	<ul style="list-style-type: none"> • The activity supports multisensory learning (objects can be looked at, described orally, touched, and manipulated) • The activity proposes work in pairs or groups (mixed ability groups, collaboration of non-special needs and special needs children is recommended), promoting mutual understanding and respect • The activity can be modified according to the needs, interests, and abilities of all learners in the classroom
Content related anchor points	<p>Lesson plans developed from this concept can be used to work on:</p> <ul style="list-style-type: none"> • Measuring of length, area, volume, and/or mass • Unit conversions

Methodology related anchor points	<ul style="list-style-type: none"> • Find a topic of interest, if possible one from everyday life – measuring “a (general abstract) rectangle” can be boring for students • Find tasks (as in “things to do”) for everyone in the classroom • Start with (mathematical) tasks that have a simple answer • Use real objects (manipulatives) that students can look at, touch, and work with if at all possible • If feasible, let students describe similar objects that they have seen before (i.e. relate to previous experiences of students)
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Ideas for lesson plans

The lesson can start by introducing the object (if possible in real life, or else a depiction of it) to be measured. It would be good to relay a context from everyday life, i.e. explaining why this object needs measuring, and what needs to be measured. This might be the mass of recipe ingredients, the area of the classroom walls that need painting, the area of the sports grounds where new grass needs to be grown, the length of a fence that needs to be built, the size of a box where all the students' math books can fit in etc. There may be a short period where students can tell their experience with similar situations.

Students should then form (if possible, mixed needs) groups, and the teacher hands out a measuring task (and, if special measuring instruments are needed, also the required tools). The instructions of the task should be as specific as possible, i.e. it should be made clear what has to be measured with which tool(s). The groups could either all receive the same task (and then compare their answers with the other groups), or they could receive different tasks (and then present their task and result to the whole class). The first task should be a simple measuring task with no conversions necessary.

The student groups are then given subsequent task(s) that can be more complex and may require multiple measurements, unit conversions, and/or mathematical calculations to get to the desired result. For the more complex task(s) it is probably better if all groups receive the same one(s).

The lesson should in any case end with all groups coming together again, and each group (or, if time constraints do not allow this, a few selected groups) presents their result. It would be good to relate the results back to the original context of the problem.

Comment: Discussions about the exactness of measurements could ensue – they can be an important learning tool, but should not end in a “yours is better, yours is worse” declaration by the teacher.



AKTION ÖSTERREICH – TSCHECHISCHE REPUBLIK
AKTION ČESKÁ REPUBLIKA – RAKOUSKO

Inclusive teaching units in mathematics
Inkluzivní výukové jednotky v matematice

Concept of teaching material

Title	Discounts and sales
Teaching unit	Let's go shopping
Age of students/grade	7 th grade
Length of the unit	1 - 2 x 45 minutes
Mathematical topic	percentages
Prerequisite knowledge	<ul style="list-style-type: none"> • Basic calculations with percentages
Materials needed	<ul style="list-style-type: none"> • Supermarket fliers
Objectives	<p>The pupil</p> <ul style="list-style-type: none"> • can apply mathematics in real-world problems with percentages • has improved pupils' financial literacy • understands the concept of discount in a shop • can analyze data in a supermarket flyer • cooperates in pairs and a group of students
Notes on work with special needs children	<ul style="list-style-type: none"> • The activities are multisensory – the pupils work with fliers, produce fliers, move around the classroom, cooperate in groups. • Pair work allow special needs pupils to cooperate with intact pupils. This promotes their mutual understanding, respect of each other. Team skills are developed.
Content related anchor points	<p>Lesson plans developed from this concept can be used to work on:</p> <ul style="list-style-type: none"> • percentages • rule of three • base • development of financial literacy
Methodology related anchor points	<ul style="list-style-type: none"> • Use of simpler tasks with number answers (this will allow special needs pupils to succeed.) • For more complex tasks, work in pairs and groups, use mixed pairs (intact and special needs pupils) • Allow sharing everyday experience in the initial phase of the lesson (shopping, sales seasons etc.) • Use visual aids • Check understanding

Ideas for lesson plans

Any tasks in which pupils calculate the discount of an item belongs here. Either they know the original price and the discount in percentages and calculate the final price, or they check in a supermarket flyer that there are no mistakes in the prices, or they know the original price and the price after discount and then they state the percentage of the discount, or they know the price of the item after discount and know the discount in percentages and find the original price...

Role play is welcome here. Also use of the Storypath method, when a group of friends/a family meet to discuss what to buy for the next week and where it to go shopping for good prices.

An option is to provide the price of the same item in different shops (the original price and the discount) looking for where to buy the item at the best price.

The pupils can develop their own supermarket flyer. This can be a longer project in which teams first research the prices of selected items in some supermarket and then make a flyer with discounts. Pupils can also select specific items, look at their prices in several shops and express in per cents the differences in their prices. This data can be presented in tables, i.e. the pupils develop their skills in handling and presenting data.