

# Final Report for the Project “Analysis of multicomponent fluid dynamical equations”

## 1. ACTIVITIES

The aim of the bilateral project was to enhance the doctoral training of the PhD students in Prague and Vienna. This training was performed on three levels:

- Teaching: a) A mini-course in Prague was presented by Dr. Nicola Zamponi (Vienna) on the derivation of hydrodynamic models from the Boltzmann equation. The topic of the mini-course was new and therefore beneficial to all PhD students in Applied Mathematics in Prague. b) A mini-course in Vienna was presented by Doc. Milan Pokorný on the mathematical theory of compressible Navier–Stokes equations. The topic is not so much studied in Vienna, therefore it was interesting for PhD students in Vienna.
- Scientific presentations: Czech and Austrian researchers have given scientific talks, particularly intended for PhD students. The talks in Vienna took place in the PhD seminar of the doctoral school and in Prague as part of the Seminar on PDEs. For details, see the list below.
- Scientific work: The scientific work was concerned with the modeling and the analysis of fluid mixtures. Professors, postdocs, and PhD students from Vienna and Prague worked together. Currently, a scientific article involving professors and postdocs is in progress.

## 2. RESULTS AND EVALUATION

The research involved compressible Navier–Stokes–Maxwell–Stefan systems. We have developed the first steps of an existence analysis, which have been summarized in the following draft:

- M. Bulíček, A. Jüngel, M. Pokorný, and N. Zamponi. Existence analysis of a compressible fluid model for heat-conducting and chemically reacting mixtures. Draft, 2017.

A compressible fluid model, consisting of the Navier–Stokes–Fourier equations and Maxwell–Stefan type fluxes, is analyzed. The model is derived in a thermodynamical consistent way from the Helmholtz free energy and the constitutive equations. The free energy is chosen such that appropriate bounds on the temperature can be achieved. Currently, a priori estimates independent of the approximation parameters have been derived. The

existence of approximate solutions and the compactness results are under investigation. This work extends the results obtained recently by Druet, Dreyer, Gajewski, and Gühlike in the isothermal setting (preprints at WIAS Berlin, Germany, 2017).

We will proceed to complete the existence analysis in 2018–2019 in the framework of the recently accepted bilateral project “Mathematical modelling and analysis of chemically reacting electrically charged complex fluids”, supported by ÖAD Austria.

The following scientific presentations have been given:

- M. Bulíček (Charles University). Variational integrals with linear growth and limiting strain models. TU Wien, 21 Nov 2017.
- A. Jüngel (TU Wien). Multi-species systems in biology: cross-diffusion and hidden gradient- flow structure. Seminar on PDEs, Czech Academy of Sciences, Prague, 10 May 2016.
- A. Jüngel (TU Wien). Analysis of cross-diffusion systems arising in nonequilibrium thermodynamics. Roztoky/Prague, 03 Aug 2017.
- O. Leingang (TU Wien). Discrete blow-up behavior for the Keller-Segel system. Seminar on PDEs, Czech Academy of Sciences, Prague, 30 Aug 2017.

Moreover, as promised in the project proposal, the following mini-courses were presented by Dr. Nicola Zamponi (Vienna) at the Charles University Prague and by Doc. Milan Pokorný (Prague) at the Technical University in Vienna:

- N. Zamponi. From the Boltzmann equation to hydrodynamic models. Charles University, Prague. Duration: 16 hours, 16–26 October 2017.
- M. Pokorný. Compressible Navier–Stokes equations. Technical University Vienna. Duration 16 hours, 13–16 March and 24–28 April 2017.

The scientific results show that the project was conducted very successfully. The mini-courses and the scientific lectures contributed to the PhD training in Prague and Vienna. The presentations of the Czech scientist was given at the “PDE Afternoon” of the doctoral school “Dissipation and Dispersion of Nonlinear Partial Differential Equations” and about 50 participants (most of them were PhD students) attended the lecture. The presentation of the Austrian participants were given as part of the Seminar on PDEs which is traditionally held at the Mathematical Institute of the Czech Academy of Sciences and is attended by

scientists from this institute, from the Charles University and by the PhD students. The mini-courses of the Viennese participant Dr. Zamponi and the Prague participant Doc. M. Pokorný complemented the PhD training in Prague and Vienna by new and modern scientific topics.

### 3. PARTICIPATING RESEARCHERS

Austrian participants:

- Senior Researchers
  - Professor: Ansgar Jüngel, TU Wien.
  - Postdoctoral researcher: Nicola Zamponi, TU Wien.
- PhD students
  - Anita Gerstenmayer, TU Wien.
  - Oliver Leingang, TU Wien.

Czech participants:

- Senior Researchers
  - Doc. Milan Pokorný, Charles University, Prague.
  - Dr. Miroslav Bulíček, Charles University, Prague.
- PhD students
  - Erika Maringová, Charles University, Prague.
  - Michal Bathory, Charles University, Prague.

### 4. SIGNATURES



Doc. Mgr. Milan Pokorný, Ph.D. (Prague)



Prof. Dr. Ansgar Jüngel (Vienna)