

## ***Final report AKTION 73 p 12***

**Title: Zolotarev polynomials and their applications in spectral analysis and filter design**

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### **Partner in co-operation:**

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### **Workshop realization and project solution:**

#### **Workshop realization:**

Two workshops were realized

1. Workshop 1: 17<sup>th</sup> - 20<sup>th</sup> of October 2015 in Prague
2. Workshop 2: 11<sup>th</sup> – 13<sup>th</sup> of July 2016 in Wels

#### **Project solution and modification**

Two 4-days workshops initially given in the project were slightly modified. First workshop “Workshop 1” was held as planned, but the second workshop “Workshop 2” was amended as follows. Due to enormous workload, one member of the CTU staff could not travel to Wels as expected. Prof. Vlček had become a vice-rector of CTU and he had to coordinate a new project preparation. Therefore only Prof. Zahradník and his colleague Ing. Šusta visited School of Engineering and Environmental Sciences in Wels. For this reason, the duration of the second workshop was shortened to three days instead of planned four days. But it is necessary to point out, that all proposed topics of the project were fulfilled. The reason is that the first workshop in Prague was very intensive and involved more issues and activities than expected.

The primary output of the 1st workshop was the detail acquaintance with research topics solved in both institutions (CTU and UAS) and the selection of joint items for next research. Moreover, we extensively discussed possible modes of next co-operation including the definition of the frame of bilateral project and publications. The output of the second workshop was the preparation of a new joint publication on Zolotarev polynomials (ZP)

which more complies with research specialization of both groups at CTU and UAS. The finalization of the grant proposal was deferred until all publications are submitted. Another reason is to establish new connections with other workers in the area of Zolotarev polynomials. Both steps might increase the probability of successful getting international project.

### **Results of this project:**

1. Two realized workshops with primary results: detailed acquaintance with activities and research at Wels and Prague, the preparation of next close cooperation and projects including joint publications.
2. One Ph.D. thesis was submitted to defense preparation process by Václav Turoň. The thesis completion was finalized thanks to the help of prof. Schiefermayr during his stay in Prague.
3. One publication was submitted to impacted journal (now it is under a review process); its topic is the numerical behavior of methods for Zolotarev polynomial generation (corresponding author Jan Kubák). The critical points of both the Ph.D. thesis and the paper were discussed with Prof. Schiefermayr during his stay in Prague.
4. Two others papers on Approximate Discrete Zolotarev transform (corresponding author R. Špetík) and ADZT implementation in real time on logic gate arrays are close to completion (corresponding author Jan Kubák). Also, problems and open questions were consulted with prof. Schiefermayr.

Both workshops were very useful also for two Ph.D. students who are focused on ZP and their applications. Our research of ZP and their use for the analysis of non-stationary signals and digital filter design was extended, and new results have been achieved. The very interesting result is that symmetrical Zolotarev polynomials and their spectra can be generated up to degree 2000 without the loss of precision. These types of Zolotarev polynomials are extremely useful for both the filter design and spectral analysis. Another new and exciting result is that Zolotarev spectrum of the mixture of sinusoids involves information not only about frequencies but also the information about the instantaneous envelopes of all possible couples of these sinusoids. Also very useful is the extension of Zolotarev polynomials to the filter bank concept. On the other hand, the discussions contributed to findings that suggested Zolotarev transform in its present implementation is not suitable for lossy compression as expected before. All these achievements were strongly accelerated by the cooperation enabled by this project. This project brings personnel meetings, and very productive discussions face to face. Results of the proposed project fall into the area of mathematics, signal analysis, digital filter design and biomedical engineering. Thus we can conclude that the synergic effect from our tight cooperation was as we expected.

### **Detail description of realised workshops and their results.**

#### **Workshop 1 – Action 1:**

This workshop in Prague in October 2015 was focused on the presentation of achieved results, and on the discussion about possible further work. We also start the discussion about possible bilateral projects and further contacts. We have got acquainted with research topics solved in both institutions (CTU and UAS). The areas of common interests have been found.

#### **Working schedule of the 1st Workshop**

##### **Day 1**

Arrival and accommodation

##### **Day 2**

Topic: Presentation of Zolotarev polynomials (ZP) and Approximated Discrete Zolotarev Transform (ADZT)

Venue: room # 530

Department of Circuit Theory, Czech Technical University in Prague, Faculty of Electrical Engineering

Content of the workshop:

Presentations:

Klaus Schiefermayr: Characterization of Zolotarev polynomials with the help of Jacobi's elliptic and theta functions, Radim Spetik: From Zolotarev polynomials to ADZT, Vaclav Turon: ADZT properties and possible applications, Pavel Masa: ADZT and signal reconstruction, Jan Kubak: Alternative approach to Zolotarev and new progress in building DZT in time domain, selected problems and behavior of the DZT transform, selectivity of ZP with respect to the degree of non-stationarity,

Discussions:

Discussion conducted by prof. Vlček and prof. Schiefermayr about possible co-operation on the paper "Algebraic Solutions for Polynomials Defined on Two Disjoint Intervals" and the discussion about expected features of Zolotarev polynomials which are still not completely clear including the orthogonality of ZP. Detail discussion of V. Turoň and J. Kubák with prof. Schiefermayr about topics of their PhD study. Especially, in this point the stay of prof. Schiefermayr proved to be very useful and encouraging.

### **Day 3**

Topic: Application of Zolotarev polynomials to digital filter design and

Venue: room #505d,

Department of telecommunications, Czech Technical University in Prague, Faculty of Electrical Engineering

Content of the workshop:

Presentations:

Klaus Schiefermayr: Zolotarev polynomials on several intervals, Pavel Zahradnik: Application of Zolotarev polynomials in the digital filter design, Michal Šusta: Numerical evaluation of Zolotarev polynomials using theta functions

Discussions:

Discussion about research topics of M. Šusta.

Other activities: Campus Tour guided by International Office

### **Day 4**

Discussion:

Venue: room #523, Department of Circuit Theory, Czech Technical University in Prague Faculty of Electrical Engineering

Topics: discussion about possible modes and topics of next co-operation.

Discussion about open questions on ZP and about two papers being prepared by J. Kubák.

Discussion about selected topics from PhD thesis of V. Turoň.

Continuation of discussions during working lunch near the FEE campus.

### **Day 5**

Departure.

### **Costs for workshop 1:**

4-day stay of 1 UAS staff at CTU Prague 6000 Kc.

Meal allowance in ČR (Kč): 4-day stay of 1 UAS staff at CTU Prague 2000 Kc.

### **Workshop 2 – Action 2:**

The second workshop at Wels in August 2016 was focused on the presentation of new results achieved at UAS and CTU. During this workshop start discussion about another joint publication on Zolotarev

polynomials.

### **Working schedule of the 2<sup>nd</sup> Workshop**

#### **Day 1**

Arrival and accommodation.

#### **Day 2**

Topics: New results in time-domain DZT, filter design and robust evaluation of ZP.

Presentations:

Klaus Schiefermayr: Zolotarev polynomials and inverse polynomial images,

Pavel Zahradnik: New perspectives and problems in using Zolotarev polynomials in the filter and filter bank design. Michal Susta: Notes on robust evaluation of Zolotarev polynomials

#### **Day 3**

Discussion about other next possible modes of co-operation (including joint diploma a PhD theses), about new joint publication and joint projects (including European ones).

The frame of a grant proposal has been partially prepared but the detail preparation of a grant has been postponed until all planed publications are submitted to increase the chance of project ziskani.

Discussion about selected research topics of M. Šusta. Discussion about the improvement of mathematical description of Zolotarev polynomials and at the same time on investigation of new applications of ZP to filtering and spectral analysis.

Other activities: Campus Tour

Departure.

#### **Costs for workshop 2:**

Travel costs for 2 members of academic staff were 3000 Kč

3-day stay and meal allowance for 2 CTU staff were 368 EUR

Remaining financial support from DZS – AKTION will be returned

Prague - Wels, August 21, 2016

  
Pavel Sovka and Klaus Schiefermayr