## Final Report for project AKTION 51p7

## **General Report**

Within the framework of project AKTION Austria - Czech Republic, Nr. 51p7, Ing. Luboš Střelec has visited Linz in these periods: 28.7.-1.8.2008, 22.10.- 28.10.2008, 25.5.-27.5.2009, 8.6.-10.6.2009, 17.6.-19.6.2009 and 27.7-30.7.2009. Dr. M. Stehlík has accomplished 1 trip from Linz to Brno in period 19.11.-20.11.2009. Unfortunately, Dr. Adamec was was not able to accomplish the planned visit.

### Planned activities

In the literature up to now only partial answers on the question on favorable normality testing procedures for heavy tailed alternatives can be find. The systematic approach was still missing. To tackle this problem was main aim of the AKTION 51p7.

Consequently, the aim of this project was to provide the through and systematic study of the testing procedures for the normality based on the robust variants of the moments. In this project we planned to provide a systematic study of a class of tests RTJB(i,j,k,l) and RTRJB(i,j,k,l) (introduced by both principal investigators during the AKTION stay of Ing. L. Strelec in Linz in 2008) and to compare them with the other existing procedures, e.g. the Jarque-Bera test, the robust Jarque-Bera test, the Shapiro-Wilk test and Medcouple tests etc. We planned to study the behavior of power functions for different alternatives in small and moderate samples. The applications in real data situations with some general conclusions were planned in some important areas, i.e. a) financial time series, b) testing for the lactate curves for athletes, c) operational risk assessment and d) data from tourism sector. Finally, project was designed to be a natural continuation of work started by the AKTION Stipendium in February 2008 "Verification of the robust Jarque-Bera test". During this stay Ing. Strelec spent a month in Linz and have been conducting with Dr. M. Stehlík an introductory research, which has been looking to had a potential to develop in a more systematic way. We expected at least two publications in a good level scientific journal accompanied by a couple of research reports and proceeding papers.

#### **Obtained Results**

Research in AKTION 51p7 focused mainly on the theoretical aspects of robust testing for normality. We have defined RT class of robustified Jarque-Bera tests based on the relaxation of the form of moment estimator. We have finalized the following tasks:

In [1] we modify the classical Jarque-Bera test and the robust Jarque-Bera test of normality. We use the median as an estimator instead of the mean in the classical Jarque-Bera test and in the robust Jarque-Bera test. This leads to the modified Jarque-Bera test and the modified robust Jarque-Bera test. Paper also demonstrates results of simulation studies of power of such tests with the various alternatives - light tailed alternatives as exponential, lognormal and gamma distributions, heavy tailed alternatives as Cauchy, Laplace,  $t_3$ ,  $t_5$  and logistic distributions and short tailed alternatives as beta and uniform distributions. These tests of normality are also used for normality testing of selected datasets of financial time series. Source data include logarithmic returns of monthly average prices of Prague stock exchange index PX and monthly average prices of CZK/EUR exchange rate in the period from 2000 to 2007.

In [2] we compared power of selected tests of normality - the Shapiro-Wilk test, the classical Jarque-Bera test, the robust Jarque-Bera test, the Lilliefors (Kolmogorov-Smirnov) test and

the Medcouple test. Consequently, paper demonstrates results of simulation studies of power of such tests for the various alternatives as heavy tailed alternatives (Cauchy, Laplace,  $t_3$ ,  $t_5$ ,  $t_7$  and logistic distributions), light tailed and very short tailed alternatives (exponential, lognormal, gamma, beta and uniform distributions) and contaminated normal distributions simulated outlier values.

In [3] and [5] we discuss effects of deviations from hypothetized normality. Two models are considered, one is the first pension pillar (and we consider here very small samples, which plays some role at start of some pension system or at early phases of it) and second one of modeling for IBNR (here we consider mid-samples). We will show that at early phases of 1st pension pillar in Slovakia the estimation of upper probability of oversizing of critical constant given by Potocký and Stehlík (2005) fits well. For the case of IBNR reserves, the date given by Stelljes (2006) are significantly more skewed and thus further research is needed for appropriate modelling of these reserves.

In [4] we demonstrate results of simulation studies of power of some new tests of normality used medcouple as a robust measure of skewness compared with the classical Jarque-Bera test and the robust Jarque-Bera test for the various alternatives as heavy tailed alternatives (Cauchy, Laplace,  $t_3$ ,  $t_5$ ,  $t_7$  and logistic distributions), light tailed skewed alternatives (exponential, lognormal and gamma distributions), very short tailed alternatives (beta and uniform distributions) and contaminated normal distribution simulated outlier values.

In [6] we introduce a general form of the robust Jarque-Bera test to systematize the results from some recent studies on variants of Jarque-Bera tests and give general guidelines for appropriate small sample testing for normality. We prove the asymptotical normality of introduced robust measures of skewness and kurtosis, together with the consistence of given tests. The introduced test statistics have asymptotically  $\chi^2$  distribution, as does the Jarque-Bera statistic. Our tests are robust and have higher power than the medcouple tests and classical Jarque-Bera test. The introduced general class of robust tests of the normality is illustrated with selected datasets of financial time series.

In [7] we demonstrate some results of simulation power studies of selected tests for normality - the Shapiro-Wilk test, the Lilliefors test, the classical Jarque-Bera test, the Jarque-Bera-Urzua test, the robust Jarque-Bera test, the directed SJ test and five versions of the Medcouple test - for a heavy tailed alternatives (Cauchy, Laplace,  $t_3$ ,  $t_5$ ,  $t_7$  and logistic distributions) and ARMA, ARCH and GARCH processes which are usually used for financial time series analysis. These tests of normality are also used for normality testing of selected datasets of financial time series. Source data include logarithmic returns of average prices of selected stock exchange indexes (DJI and PX) and price of exchange rates (CZK/EUR and CZK/USD) in the period from 1995 to 2008.

In [8] we introduce the location functional based form of the robust Jarque-Bera (JB) test to systematize the results from some recent studies on variants of JB tests and give general guidelines for appropriate small sample testing for normality. Particularly, the special cases of this class are the classical Jarque-Bera test, the Jarque-Bera-Urzua test, the robust JB test introduced by Gel and Gastwirth (2008), the robust directed test against heavy tails by Gel, Miao and Gastwirth (2007), the Geary's test and Uthoff's test. Then we concentrate on the tractable subclass RTJB of robustified JB tests which substantially improve the properties of classical JB test. We prove the asymptotical normality of introduced measures of skewness and kurtosis, consistency, and asymptotical distribution of tests. We provide guidelines for robust testing for normality and illustrate new procedures on a selected financial dataset.

# Synergies of the project

Dr. Milan Stehlík has visited Ceské Budějovice on 5th December 2008 and on 6th December 2008 to find a new contacts for further applications of derived methods. This finally led to the accepted project Project 53p19 "AKTION Czech Republic-Austria": Spatial mapping of ticksand tick-borne infectious diseases of the region of South Bohemia and Upper Austria. We will apply some results from project 51p7, especially those on robust testing for normality against heavy tailed alternatives also in the project 53p19.

**Publications of the project:** There are 8 publications which have been made with the help of the project AKTION 51p7. They are listed in the following References section.

# References

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